Probabilistic Modeling of State Transitions on the Self-Organizing Map: Some Temporal Financial Applications

Peter Sarlin | Zhiyuan Yao | Tomas Eklund

TUCS Technical Report
No 1023, October 2011
Probabilistic Modeling of State Transitions on the Self-Organizing Map: Some Temporal Financial Applications

Peter Sarlin
Åbo Akademi University, Department of Information Technologies

Zhiyuan Yao
Åbo Akademi University, Department of Information Technologies

Tomas Eklund
Åbo Akademi University, Department of Information Technologies

TUCS Technical Report
No 1023, October 2011
Abstract

Self-organizing maps (SOMs) have commonly been used in temporal financial applications. This paper enhances the SOM paradigm for temporal data by presenting a framework for computing, summarizing and visualizing transition probabilities on the SOM. The framework includes computing matrices of node-to-node and node-to-cluster transitions and summarizing maximum state transition. The computations are visualized using feature plane representations. The future state transitions can also be used for finding low- and high-risk profiles as well as for assessing the evolution of probabilities over time, where the cluster centers express the representative financial states while the probability fluctuations represent their variation over time. We demonstrate the usefulness of the framework on two previously presented SOM models for temporal financial analysis: financial benchmarking of banks and monitoring indicators of currency crises.

Keywords: transition probabilities, Self-Organizing Map (SOM), temporal data, financial data, financial institutions, currency crises
1. Introduction

Today’s decision makers are often faced by enormous amounts of financial data available for decision making purposes. Access to online financial databases, such as Thomson One, Amadeus and Bankscope, can provide nearly endless amounts of multivariate financial time-series data. However, due to non-linear relationships, high dimensionality and non-normality often inherent in financial data, utilizing these data can be a significant challenge for traditional statistical tools and spreadsheet programs. Instead, various data mining and pattern recognition tools have been applied for this purpose.

One potential tool is the Self-Organizing Map (SOM) [1–2], an unsupervised neural network-based projection and clustering tool often used for exploratory data analysis. Although most of the early SOM applications have been in the area of medicine and engineering [3], the SOM has also been used in a large number of financial applications [4], including financial benchmarking [5–6], bankruptcy prediction [7–8], financial crisis monitoring [9–10], economic welfare analysis [11], customer churn analysis and segmentation [12–13], and stock price forecasting [14–15], just to name a few.

The general SOM paradigm is an ideal tool for building visualization systems; however, manually identifying the positions and patterns in a SOM model is not necessarily a simple process. As the applications above illustrate, financial data typically belong to a time series. Variations of the SOM algorithm itself, using delayed or reinforced learning, have been proposed for dealing with temporal data by using leaky integrators or recurrent networks [16–18], for example. In Sarlin and Eklund [19–20], the SOM was extended to show membership degrees of each time-series point to each cluster using fuzzy C-means clustering as well as using a distance-based metric. This is suitable for assessing the current state, but says nothing about future transitions. Hence, a method for illustrating state transitions is required. Most often, trajectories [21] have been used in the SOM literature to illustrate these state transitions [e.g., 6–7,22]. While state transition patterns require a large number of observations for significance, trajectories can only be used on a limited set of data in order not to clutter the display. Thus, they provide no overall information about trends in the dataset. For finding these patterns, be they cyclical or not, the switches should be summarized from transition probabilities, something that is not apparent from studying the elements of the SOM units. Transition probabilities reveal the strengths and actual directions of the patterns as well as tolerates partial truth, uncertainty and imprecision by being both entirely data driven and probabilistic in nature.

Hybrid approaches combining standard SOMs with machine learning classification techniques such as neural networks and support vector machines [14–15], have been proposed for modeling future state transitions, but these suffer from high complexity and computational cost as well as impaired visualization capabilities. Methods to directly deal with state transitions in SOMs have been introduced in previous applications, such as by using hidden Markov models [23] and one-level node-to-node transition probabilities [24–26]. Here, we propose a framework that significantly further enhances the visualization and exploitation of transitions probabilities in previous applications. As we show state transitions on a two-level clustering, we enable discovery of detailed node-to-node and node-to-cluster patterns. Differences in transition probabilities between nodes within the same clusters reveal information since a homogeneous cluster does not necessitate similar state transitions. We also emphasize a user-oriented and easily interpretable visualization of the transition probability matrices; not only static visualizations but also the evolution of and reaction to transition probabilities over time. Finally, this framework can as well be used for company or country profiling by presenting low- and high-risk mean profiles. The contribution of
this paper is a novel framework for computing, visualizing and exploiting transition probabilities on a two-level SOM clustering.

The SOM has been shown to be a particularly feasible tool for building visual monitoring systems for financial benchmarking of companies [5–6,20,27] and for monitoring indicators of country-level financial crises, such as currency, debt and systemic crises [9–10,22,28]. We show the added value of the transition probability framework by applying it to two of these SOM models: financial performance analysis of banks [20] and monitoring indicators of currency crises [22]. As Minsky’s [29] and Kindleberger’s [30] vindicated financial fragility view of a credit or asset cycle, transitions in currency crisis indicators can as well be justified according to the stages of the Kindleberger-Minsky model [31]. Similarly, fluctuations in performance of financial institutions justify transitions of firm-level data.

The paper is structured as follows. Section 2 introduces a framework for transition probabilities on the SOM. In Section 3, the framework is applied on financial time series, while Section 4 concludes by presenting our key findings.

2. Methodology

2.1. Self-Organizing Maps

The SOM is a method with simultaneous clustering and projection capabilities first developed by Kohonen [1]. As the SOM algorithm is well-known and the main emphasis is on transitions on the SOM, we do not present details of it here – for further reference see [2]. The Viscovery SOMine 5.1 package is used in this study mainly for its excellent visual representation. The training process starts with a linear initialization of the reference vectors. The first step compares all input data vectors \( x_j \) (where \( j=1,2,\ldots,N \)) with the network’s reference vectors \( m_i \) (where \( i=1,2,\ldots,M \)) to find the best match \( m_u \):

\[
\| x_j - m_b \| = \min_i \| x_j - m_i \| \tag{1}
\]

Then the second step adjusts each reference vector \( m_i \) with the batch updating formula [2]:

\[
m_i(t+1) = \frac{\sum_{j=1}^{N} h_{i.w(j)}(t) x_j}{\sum_{j=1}^{N} h_{i.w(j)}(t)}, \tag{2}
\]

where \( t \) is a discrete time coordinate and \( h_{i.w(j)} \) a decreasing function of neighborhood radii and time.

As the map describes a multidimensional space on a two-dimensional grid of nodes, all information on the map cannot be visualized in two dimensions. To further enhance the visualization, the variables are separately shown on their own grids. Each feature plane displays the distribution of that variable on the map, with cold colors (blue) indicating low values and warm (red) indicating high values. As the feature planes are different views of the same map, one unique point represents the same node on all planes. Thereby, the characteristics of the SOM model can be identified by studying the underlying feature planes.

The nodes of the map can further be divided into clusters of similar nodes. We use hierarchical clustering with the following modified Ward’s [32] criterion as a basis for merging two candidate clusters:

\[
d_{kl} = \left\{ \begin{array}{ll}
\frac{n_k n_l}{n_k + n_l} \left\| k - c_l \right\|^2 & \text{if } k \text{ and } l \text{ are adjacent} \\
\infty & \text{otherwise}
\end{array} \right., \tag{3}
\]

where \( k \) and \( l \) represent clusters, \( n_k \) and \( n_l \) the cardinality of clusters \( k \) and \( l \), and \( \left\| k - c_l \right\|^2 \) the squared Euclidean distance between the cluster centers of clusters \( k \) and \( l \), and the distance between non-adjacent clusters is infinite. When clusters \( k \) and \( l \) are merged to cluster \( h \), the cardinality is the sum of the cardinalities of \( k \) and \( l \) and the centroid the
mean of $c_k$ and $c_l$ weighted by their cardinalities.

2.3 Transition Probabilities on the SOM

Transition probability matrices (TPMs) produce a probabilistic model of the temporal variation in a SOM model. The two-dimensional grid of Section 2.1 is used to compute probabilities of switching from each node to a specified region in a specified time period, where the location per time unit is derived using Eq. 1 (vectors $x_i$ and $m_i$ consist of as many dimensions as data). First, we compute for each node $m_i$ the probability of transition to every other node $m_b$:

$$p_{ib}(t+s) = \frac{n_{ib}(t+s)}{\sum_{b=1}^{M} n_{ib}(t+s)} , \quad (4)$$

where $n_{ib}$ is the cardinality of data switching from $m_i$ to $m_b$, $t$ is a time coordinate and $s$ is the time span for the switch. In other words, the transition probability $p_{ib}(t+1)$ equals the cardinality of transitions from node $m_i$ to node $m_b$ divided by the sum of transition from node $m_i$ to each node $m_{1,2,...,M}$. On a SOM grid with four nodes, this could in practice mean that for, say, node $m_1$ the probability of being in period $t+1$ in $m_{1,2,3,4}$ could be 0.5, 0.2, 0.2 and 0.1, respectively. Formally, a TPM corresponds to maximum likelihood estimates of the switches or a first-order Markov model. It can, however, be computed for different time spans, as appropriate, and summarized to switches between clusters or any other region on the map. For example, node-to-cluster switches are computed using $p_{il}$, where the transition refers to movements from reference vector $i$ to cluster $l$ (where $l=1,2,...,C$), thus:

$$p_{il}(t+s) = \frac{n_{il}(t+s)}{\sum_{l=1}^{C} n_{il}(t+s)} . \quad (5)$$

For correcting coincidental results due to e.g. lack of data, the TPMs $p_{ib}$ (as well as $p_{il}$) can be computed as an average of several $s$ values (where $s=1,2,...,S$):

$$p_{il}(t+[1,2,...,S]) = \frac{\sum_{l=1}^{S} n_{il}(t+s)}{\sum_{s=1}^{S} n_{il}(t+s)} . \quad (6)$$

To sum up, we propose the following three computations:

1. TPMs for node-to-cluster switches ($p_{il}(t+s)$) as in Eq. 5) for a specified set of $s$ values.
2. Summarize the TPMs from Step 1 by computing to which cluster $l$ an observation in $m_i$ is most likely to switch and with what likelihood, i.e., showing maximum transition probabilities ($\text{max}_l(p_{il})$) conditional on switching. This combines the direction and strength of the probability into a vector.
3. For summarizing the computations in Steps 1 and 2 over time, compute average transition probabilities over a chosen set $s$ values ($p_{il}(t+[1,2,...,S])$ as in Eq. 6).

Transition probabilities for nodes can be visualized on feature planes, where one unique point represents the same node on the previously presented SOM grid. Thereby, the structure of the transitions on the SOM model can be directly identified by studying the underlying transition probability feature planes. The above computations are represented using the following three feature plane visualizations:

1. Show the probability to transit to a particular cluster for each node on own feature planes, such that the color code of each node represents its probability to transit to that particular cluster.
2. Summarize the information in Step 1 to one feature plane, where the color code in each node is the probability of the most likely switch and a label
represents that cluster (non-transiting nodes are left empty).

3. Create the same feature planes as in Step 1 and 2, but as an average over a chosen set of $s$ values.

To normalize the color scales for different cluster sizes, but still show differences over time, we set the color scales of the feature planes for all $s$ values and sets of $s$ values as to that for $s=1$ (i.e. $t+1$). The temporal dimension of a bank or country can as well be represented by associating each time-series point with the transition probability of its BMU. This enables a line graph representation of the state switch probabilities over time, where cluster centers are representative financial states and the variation in transition probabilities indications of future financial performance. The transition probabilities are used for country profiling by presenting low- and high-risk mean profiles.

### 3. Applications of the Transition Probability SOM on Financial Time Series

#### 3.1. Financial Benchmarking Model

The financial benchmarking model was created as a complement to the ongoing EU stress testing of the European banking sector, on account of the fallout from the recent financial crisis. The data were retrieved from the Bankscope financial database, and consisted of 24 financial ratios for 855 European banks, and covered annual data for the period 1992–2008, resulting in a total of 9,655 rows of data. The SOM model was created by first applying PCA to obtain seven subdimensions of financial performance: capital ratios (PC1), loan ratios (PC2), profitability (PC3), interest revenue (PC4), non-operating items (PC5), subordinated debt (PC6) and loan loss provisions (PC7). In general, high values are better, with the exception of loan ratios, which reflect the inverse of capital to net loans and the liquidity of a bank’s assets, and loan loss provisions, which measure the ratio of risk to interest rate margins. In addition to these, the ratios Tier 1 and total (Tier 1 + Tier 2) capital were associated with the trained map, as these are the most important ratios from the perspective of stress testing. Moreover, Ward’s [32] method was used for second-level clustering. For further details of the model, readers are referred to Sarlin and Eklund [20]. The SOM model, with data for Deutsche Bank and Banco Santander from 2002–2008, is displayed in Figure 1, and its feature planes in Figure 2.
The banks in clusters A, B and C can be seen as good performers, where A can be seen as the best. The banks in cluster D are average performers, as they show average values for all variables. The banks in cluster E, F and G can be seen as poor performers, where F can be seen as the poorest. Since TPMs to seven clusters impairs the interpretability of switches, and these clusters can easily be grouped by performance, we consider the best clusters as one (A, B and C), the average cluster as one (D) and the poor clusters as one (E, F and G).

3.2. Transition Probabilities on the Financial Benchmarking Model

We follow the above three-step framework when computing the transition probabilities for the financial benchmarking model. First, TPMs were computed as switches from nodes to clusters \( p_{i}(t+s) \) as in Eq. 5. Second, the direction and strength of the switches is summarized by computing maximum transition probabilities \( \max_{i}(p_{i}) \) conditional on switching. Third, we compute the above steps for three different transition time spans \( t+1, t+2 \) and \( t+3 \), and an average for \( S=3 \). A feature plane visualization of these computations is shown in Figure 3.

The transitions on the SOM reveal several interesting patterns. The poorest group (clusters E, F and G) can be seen as inherently stable, as there are few transitions from the nodes in these clusters, irrespective of the time span. The best group (clusters A, B and C), on the other hand, is less stable, and the probability of transitions from the nodes in this cluster increases with time. The average group (cluster D) is an instable transition cluster. Based on the average probabilities, we can see that companies in clusters A, F and G are quite stable, while clusters B, C and E exhibit more transitions, and thus less stability.

The difference in stability between cluster A and clusters B and C might be due to differing business activities, as the feature planes in Figure 2 show that cluster A differs from B and C primarily in capital ratios, loan interest revenue, and subordinated debt. These ratios indicate that cluster B and C are higher risk clusters than cluster A, and thus, probably more sensitive to changing business conditions, such as interest rates. For cluster B, an interesting strong cluster-to-cluster pattern is the high probabilities of movements to cluster D (average performance).

Another interesting pattern is the difference in stability between cluster E and clusters F and G. While E, F and G are quite similar clusters in terms of performance, a clear difference can be seen in the high ratio of non-operating items of cluster E. Non-operating items are items not related to ongoing, day to day operations, such as dividends, financial investments or significant write-downs, which might partially explain the unstable nature of positions in cluster E.

The line graphs in Figure 4 show a practical company-specific application of the transition probability framework. The figure shows the state transition probabilities for Deutsche Bank and Banco Santander for 2002–08. It enables assessing the evolution of probabilities over time, where the cluster centers express the representative financial states while the probability fluctuations represent their variation over time. A label above each time-series point denotes the cluster in which the company is currently located. The trajectories of the banks are shown on the SOM grid in Figure 1. Since the probability of staying in a cluster is most commonly highest, the interesting patterns are the addresses of the switches and the probability trend of the most likely switch. The transition probabilities for Deutsche Bank are dominated by two clusters, E and F. The switches between clusters E and F are thus expected, and the final trend indicates a future movement back to cluster F. For Banco Santander, the first movement from cluster G to D is unexpected, but the movement back to G is accordingly preceded by an increasing trend of the transition probability to that cluster.
Notes: On each row, the three first feature planes represent the probability to switch to the corresponding clusters (or group of clusters). The last feature plane on each row summarizes the direction and strength of the switches by showing maximum transition probabilities, where the color coding represents the probability, and the label the address of the switch.

Figure 3. Feature planes of the bank model’s transition probabilities.

Figure 4. Line graphs of the transition probabilities for Deutsche Bank and Banco Santander.
3.3. Financial Crisis Model

The currency crisis model was created for visual monitoring of currency crisis indicators. The model consists of four monthly indicators of currency crises for 23 emerging market economies from 1971:1–1997:12. The indicators included are foreign exchange reserve loss, export loss, real exchange-rate overvaluation relative to trend and current account deficit to GDP, and were chosen and transformed based on a seminal early warning system created by IMF staff [33]. This model is, however, conceptually different from the benchmarking model. Each data point has a class dummy indicating the occurrence of a crisis, pre-crisis or tranquil period. A crisis period is defined to occur when exchange-rate and reserve volatility exceeds a specified threshold, while the pre-crisis periods are defined as 24 months preceding a crisis and the rest of the periods are tranquil. The class labels were associated with the model by only affecting the updating of the reference vectors (Eq. 2), not the choice of the BMU (Eq. 1). The crisis model represents cyclical behavior resembling a currency crisis or financial stability cycle. Thus, the main purpose of the model is to visualize the evolution of financial indicators to assist the detection of vulnerabilities or threats to financial stability. The model is presented in detail in Sarlin [22] and a model on the same data set is evaluated in terms of out-of-sample accuracy in Sarlin and Marghescu [10]. Moreover, a stand-alone FCM clustering has been applied on a similar data set in [34].

The contribution of each input is standardized using columnwise normalization by range. However, the effects of extremities and outliers are not eliminated, since a crisis episode is per se an extreme event. The map consists of 137 output neurons ordered on a 13x11 lattice, divided into four crisp clusters representing different time periods of the currency crisis cycle. The units were clustered using Ward’s hierarchical clustering on the associated variables. The map and its feature planes are shown in Figures 5 and 6. The map is roughly divided into a tranquil cluster on the right side of the map (cluster A), a slight early-warning and a pre-crisis cluster in the lower-left part (cluster B and C), and a crisis cluster in the upper-left part (cluster D).

3.4. Transition Probabilities on the Financial Crisis Model

We again follow the three-step framework when computing the transition probabilities for the financial crisis model. The first two steps were computed as above, but the third step is computed for only one time span \((t+1)\), and three averages for \(S=\{6,12,24\}\). A feature plane visualization of these computations is shown in Figure 7.

Similarly as for the benchmarking model, the transitions on the SOM reveal several interesting patterns. The cyclical behavior
on the model makes it, however, conceptually different. The transition probabilities in Figure 7 show that the cyclicity follows the four clusters representing states of financial stability. The feature plane representation also shows that for longer averages the clusters become, obviously, less stable. The summarized feature planes reveal that most of the nodes in the tranquil cluster A switch to the early warning cluster B while a group of mid-cluster nodes have a high probability of switching back to the crisis cluster D. Similarly, nodes in cluster B adjacent to cluster A have a high probability of moving to A while those adjacent to C move to C. From the pre-crisis cluster C, the highest probabilities are to switch to cluster B and then further on to the crisis cluster D.

Figure 7. Feature planes of the crisis model’s transition probabilities

Figure 8. Student’s t-tests on profiles of high and low risk and the average country.
The transition probabilities are used for country profiling by presenting low- and high-risk mean profiles. The low-risk profile is a group of stable nodes with an extremely high probability of staying in the tranquil cluster A, while the high-risk profile is a group of nodes with the high probabilities of moving to the pre-crisis cluster C. This type of profiling is important, since finding cracks in the financial system at an early hour is important as it would allow introduction of policy actions to decrease or prevent further build up of vulnerabilities. Student’s t-tests on the high- and low-risk profiles and the average country are shown in Figure 8. For the high-risk profile, the exchange rate overvaluation is significantly larger and the export loss smaller than the average. For the low-risk profile, the current account deficit is significantly smaller and reserve loss and exchange-rate overvaluation larger than the average.

4. Conclusion

This paper enhances the SOM paradigm for temporal data by presenting a novel framework for computing, summarizing and visualizing transition probabilities on the SOM. The framework includes computing matrices of node-to-node and node-to-cluster transitions and summarizing maximum state transition. The computations are visualized using feature plane representations. The future state transitions can also be used for finding low- and high-risk profiles as well as for assessing the evolution of probabilities over time, where the cluster centers express the representative financial states while the probability fluctuations represent their variation over time. We demonstrate the usefulness of the framework on two previously presented SOM models for temporal financial analysis: financial benchmarking of banks and monitoring indicators of currency crises. In addition to transition patterns assessed for both models, we show an assessment of transition probabilities, and reactions to them, over time for two banks and conduct low and high-risk country profiling using the financial crisis model. Thus, while the information products of the standard SOM paradigm have been evaluated as superior to previously used methods [27], the framework for assessing strengths and directions of transition patterns on the SOM further enhances the usefulness of SOM-based visualization systems. In addition to financial applications, the transition probability framework could as well be applied on a broad range of other types of temporal problems. The main limitations of probabilistic modeling of state transitions are, however, the requirements of large datasets and small SOMs.

5. Acknowledgements

The financial support of the Finnish Funding Agency for Technology and Innovation (grant No. 40063/08) and the Academy of Finland (grant No. 127656) is gratefully acknowledged. We would also like to thank anonymous reviewers, who helped us to significantly improve this paper.

6. References


University of Turku
- Department of Information Technology
- Department of Mathematics

Åbo Akademi University
- Department of Information Technologies

Turku School of Economics
- Institute of Information Systems Sciences

ISBN 978-952-12-2657-1
ISSN 1239-1891