A Method for Elicitating and Combining Group Preferences for Stochastic Multicriteria Acceptability Analysis
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Abstract

Stochastic Multicriteria Acceptability Analysis (SMAA) is a family of methods to aid decision makers (DMs) in discrete decision making problems. SMAA methods are capable of handling preference information of various types. The imprecise preferences of multiple DMs can for example be represented by a joint probability distribution for criteria weights.

Dempster-Shafer theory with Analytic Hierarchy Process (DS/AHP) is a decision support method that models ignorance in the preferences of a single decision maker using Dempster-Shafer theory of belief. In this paper we show how the DS/AHP can be used to collect and aggregate the preferences of multiple DMs, and how this information can then be encoded as weight interval constraints in SMAA.

Keywords: Stochastic Multicriteria Acceptability Analysis (SMAA), preference modelling, Multiple Criteria Decision Aiding (MCDA), DS/AHP, Dempster-Shafer theory of belief
1 Introduction

Stochastic Multicriteria Acceptability Analysis (SMAA) is a family of decision support methods to aid decision makers (DMs) in discrete decision making problems with multiple decision makers. Instead of giving direct answers to the decision making problem, the SMAA methods analyze and characterize the problem, leaving the final decision for DMs. The SMAA methods have successfully been applied to real-life decision making problems (see e.g. Hokkanen et al., 1998, 1999, 2000; Lahdelma and Salminen, 2003; Lahdelma et al., 2002, 2001).

The SMAA methods allow modelling of preference information in form of probability distributions, but in practice the preference information is given in simplified form, for example, intervals for weights or ranking of criteria. When there are multiple DMs with different preferences, the preference information must be aggregated before it can be used in SMAA computations. There are multiple different ways to do the aggregation (Lahdelma and Salminen, 2001). The aggregation method may have a great impact on the results of the SMAA analysis. It is crucial that the preference information from a large set of DMs can be collected efficiently and aggregated in a consistent and theoretically sound way.

Dempster-Shafer theory with Analytic Hierarchy Process (DS/AHP) is a decision support method that models ignorance in the preferences of a single decision maker using the Dempster-Shafer theory of belief (DST). After this, DS/AHP allows the preferences to be aggregated using evidence combination according to DST (Beynon et al., 2000).

In this paper we show how the DS/AHP can be used to collect and aggregate the preferences of multiple decision makers, and how the aggregated preference information can be applied in SMAA models. We will present an analysis of a small problem using the introduced method. The article is organized as follows: We describe the SMAA methods in Section 2. The Dempster-Shafer theory of evidence is briefly introduced in Section 3, and the DS/AHP method in Section 4. In Section 5, we present how DS/AHP can be used to collect and aggregate preferences for SMAA usage, and present and analyze a small car-selection example for illustrative purposes. We end this paper with conclusions in Section 6.

2 The SMAA Methods

The SMAA-2 method (Lahdelma and Salminen, 2001) has been developed for discrete stochastic multicriteria decision making problems with multiple DMs. SMAA-2 extends the original SMAA method (Lahdelma et al., 1998) by generalizing the analysis to a general utility or value function, to include various kinds of preference information and to consider holistically all ranks. SMAA-2
applies inverse weight space analysis to describe for each alternative what kind of preferences make it the most preferred one, or place it on any particular rank. The decision problem is represented as a set of \( m \) alternatives \( \{ x_1, x_2, \ldots, x_m \} \) that are evaluated in terms of \( n \) criteria. The DMs’ preference structure is represented by a real-valued utility or value function \( u(x, w) \). The value function maps the different alternatives to real values by using a weight vector \( w \) to quantify DMs’ subjective preferences. SMAA-2 has been developed for situations where neither criteria measurements nor weights are precisely known. Uncertain or imprecise criteria are represented by stochastic variables \( \xi \) with joint density function \( f_X(\xi) \) in the space \( X \subseteq R^{m \times n} \). The DMs’ unknown or partially known preferences are represented by a weight distribution with joint density function \( f_W(w) \) in the feasible weight space \( W \). Total lack of preference information is represented in ‘Bayesian’ spirit by a uniform weight distribution in \( W \), that is, \( f_W(w) = 1/\text{vol}(W) \). The weight space can be defined according to needs, but typically, the weights are non-negative and normalized, that is; the weight space is an \( n-1 \) dimensional simplex in \( n \) dimensional space:

\[
W = \left\{ w \in R^n : w \geq 0 \text{ and } \sum_{j=1}^{n} w_j = 1 \right\}.
\]  

The value function is used to map the stochastic criteria and weight distributions into value distributions \( u(\xi, w) \). Based on the value distributions, the rank of each alternative is defined as an integer from the best rank (\( = 1 \)) to the worst rank (\( = m \)) by means of a ranking function

\[
\text{rank}(i, \xi, w) = 1 + \sum_{k=1}^{m} \rho(u(\xi_k, w) > u(\xi_i, w)),
\]  

where \( \rho(\text{true}) = 1 \) and \( \rho(\text{false}) = 0 \). SMAA-2 is then based on analysing the stochastic sets of favourable rank weights

\[
W_{i}^r(\xi) = \{ w \in W : \text{rank}(i, \xi, w) = r \}.
\]

Any weight \( w \in W_{i}^r(\xi) \) results in such values for different alternatives, that alternative \( x_i \) obtains rank \( r \).

The first descriptive measure of SMAA-2 is the rank acceptability index \( b_{i}^r \), which measures the variety of different preferences that grant alternative \( x_i \) rank \( r \). It is the share of all feasible weights that make the alternative acceptable for a particular rank, and it is most conveniently expressed percentage-wise. The rank acceptability index \( b_{i}^r \) is computed numerically as a multidimensional integral over the criteria distributions and the favourable rank weights as

\[
b_{i}^r = \int_{\xi \in X} f_X(\xi) \int_{w \in W_{i}^r(\xi)} f_W(w) \, dw \, d\xi.
\]  

The most acceptable (best) alternatives are those with high acceptabilities for the best ranks. Evidently, the rank acceptability indices are in the range [0,1], where
0 indicates that the alternative will never obtain a given rank and 1 indicates that it will obtain the given rank always with any choice of weights.

The first rank acceptability index \( b^1_i \) is called the *acceptability index* \( a_i \). The acceptability index is particularly interesting, because it is nonzero for stochastically efficient alternatives (alternatives that are efficient with some values for the stochastic criteria measurements) and zero for inefficient alternatives. The acceptability index not only identifies the efficient alternatives, but also measures the strength of the efficiency considering simultaneously the uncertainty in criteria and DMs’ preferences.

The *central weight vector* \( w^c_i \) is the expected centre of gravity (centroid) of the favourable first rank weights of an alternative. The central weight vector represents the preferences of a 'typical' DM supporting this alternative. The central weights of different alternatives can be presented to the DMs in order to help them understand how different weights correspond to different choices with the assumed preference model. The central weight vector \( w^c_i \) is computed numerically as a multidimensional integral over the criteria distributions and the favourable first rank weights using

\[
w^c_i = \int_{\xi \in X} f_X(\xi) \int_{w \in W^1_i(\xi)} f_W(w) w \, dw \, d\xi / a_i.
\]  

(5)

The *confidence factor* \( p^c_i \) is the probability for an alternative to obtain the first rank when the central weight vector is chosen. The confidence factor is computed as a multidimensional integral over the criteria distributions using

\[
p^c_i = \int_{\xi \in X: \text{rank}(i, \xi, w^c_i) = 1} f_X(\xi) \, d\xi.
\]  

(6)

Confidence factors can similarly be calculated for any given weight vectors. The confidence factors measure whether the criteria measurements are accurate enough to discern the efficient alternatives.

There are several different ways to handle partial preference information in SMAA methods (Lahdelma and Salminen, 2001). In this article we will present a novel method applying DS/AHP for collecting preference information from DMs and aggregating the information into \( L \) interval constraints for sums of weights. The constraints are given as

\[
c^\text{min}_\ell \leq \sum_{j \in C_\ell} w_j \leq c^\text{max}_\ell, \forall \ell = 1, \ldots, L,
\]  

(7)

where \( C_\ell \) is a set of criteria in constraint \( \ell \). The weight space analysis of SMAA is then performed in the restricted weight space

\[
W' = \{ w \in W | w \text{ satisfies (7)} \}.
\]  

(8)

This means that the uniform weight distribution \( f_W(w) \) is redefined as

\[
f_W(w) = \begin{cases} 
1 / \text{vol}(W'), & \text{if } w \in W', \\
0, & \text{otherwise}.
\end{cases}
\]  

(9)
3 The Dempster-Shafer Theory of Belief

In the classical Bayesian theory, the probabilities are considered to be objective. The objectivism stems from the definition of probability: the relative frequency at which an event occurs with. This type of definition does not allow modelling of ignorance, which is important especially when modelling preferences of multiple DMs in context of multicriteria decision making.

The Dempster-Shafer theory of belief extends the classical Bayesian theory of probabilities using belief functions. Instead of assigning probabilities (that sum to unity) for propositions, the belief functions assign probability masses to sets of propositions in the frame of discernment (the powerset of the set of all propositions). Let us illustrate this by an example. Consider a situation where we have to decide whether a certain chemical substance is harmful to humans. Let $p_1$ be the proposition “substance is harmful” and $p_2$ proposition “substance is not harmful”. Most people do not possess sufficient knowledge required for an informed judgment between the alternatives. According to Bayesian theory, in absence of knowledge a probability of 0.5 is assigned to both $p_1$ and $p_2$. If Dempster-Shafer theory is applied, a probability of 0 is assigned to both $p_1$ and $p_2$, and 1 to the set $\{p_1, p_2\}$ representing ignorance. Now suppose we have a third alternative, “substance is slightly harmful for humans” ($p_3$). With the modified frame of discernment the Bayesian probabilities would be $p_1 = p_2 = p_3 = 0.33$, and the DST probabilities $p_1 = p_2 = p_3 = 0$, $\{p_1, p_2, p_3\} = 1$. The Bayesian probabilities thus depend strongly on the frame of discernment, and by looking at the example it is evident that DST is more consistent in assignment of the probability masses.

Based on the previous example, it should also be noticed, that when the Bayesian approach is applied, knowledge and ignorance are indistinguishable. If DST is applied, ignorance can be modelled, and in addition certain metrics can be used for calculating the amount of total ignorance in the belief structure (Shafer, 1976).

DST terminology defines a basic probability assignment (bpa) function as a belief function that assigns probabilities that sum to unity to the propositions and sets of propositions in the frame of discernment. If there are multiple bpa’s, they can be combined using certain combination rules. The Dempster’s rule of combination is (Shafer, 1976)

$$m(\emptyset) = 0,$$

$$m(A) = \frac{\sum_{A_i \cap B_j = A} m_1(A_i)m_2(B_j)}{1 - K}, \text{ if } A \neq \emptyset,$$  

(10)

where

$$K = \sum_{A_i \cap B_j = \emptyset} m_1(A_i)m_2(B_j),$$  

(11)

$m_1$ and $m_2$ are bpa’s, $m$ is the combined bpa, and $A_1, \ldots, A_k$ and $B_1, \ldots, B_l$ are their
focal elements (subsets), respectively. This holds for all non-empty \( A \subseteq \Theta \), where \( \Theta \) is the frame of discernment.

In the rule above (10) \( K \) is the weight of conflict measuring the conflict between the two bodies of evidence. In Dempster’s rule, the weight of conflict is used for normalizing, meaning that it is distributed among all sets of propositions. This approach supposes \textit{a priori} that the ignorance can be distributed evenly. A theoretically more sound combination rule has been introduced by Yager (1987), namely Yager’s rule of combination:

\[
m(\emptyset) = 0, \\
m(A) = \sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j), \text{ if } A \neq \emptyset \text{ and } A \neq X, \\
m(\Theta) = \left( \sum_{A_i \cap B_j = X} m_1(A_i) m_2(B_j) \right) + K,
\]

where \( K \) is defined as in (10). By using Yager’s rule of combination, the weight of conflict is added to the set representing all propositions (the frame of discernment), so the conflict is added to the ignorance represented in the belief structure. We will utilize Yager’s rule when applying DS/AHP for preference combination in SMAA.

After all available evidence are combined, DST allows different characteristic values to be calculated from the bpa’s. The two most important are belief and plausibility. \textit{Belief} in set of propositions \( A \) measures the probability mass that is assigned to \( A \) or any subset of \( A \). It measures the confidence we have in \( A \), and is defined as

\[
Bel(A) = \sum_{B \subseteq A} m(B), \text{ for all } A \subseteq \Theta.
\]  

\textit{Plausibility} in set of propositions \( A \) measures the probability mass that is assigned to sets that \( A \) has common elements with. It is the amount we fail to disbelieve \( A \), and is defined as

\[
Pls(A) = \sum_{B \cap A \neq \emptyset} m(B), \text{ for all } A \subseteq \Theta.
\]  

\([Bel(A), Pls(A)]\) is thus the interval for the “true” probability of \( A \) when ignorance is taken into account.

4 The DS/AHP-method

The DS/AHP method introduced by Beynon et al. (2000) and developed further by Beynon et al. (2001) is a decision-support method utilizing the DST for preference modelling. DS/AHP is developed for decision making problems with a single decision maker, and it applies AHP (Saaty, 1980) for collecting the preferences from a decision maker, transforming them into bpa’s, and for modelling the problem as a hierarchical decision tree. AHP is a well-established method for
multicriteria decision making. In AHP, the problem is broken hierarchically into smaller pieces, until they are comparable by the DM using pairwise comparisons. From the pairwise comparisons, a comparison matrix is formed, from which criteria values for individual alternatives are obtained as eigenvectors. AHP computes also a consistency index, which measures the consistency of the pairwise comparisons.

DS/AHP differs from AHP in that it allows comparisons to be made between groups of alternatives instead of single alternatives, and that it uses Dempster’s rule of combination for aggregating the criteria instead of simple multiplications and additions. We briefly present an example for illustrating usage of DS/AHP. For a complete description of the example, see (Beynon et al., 2000). In this example, a DM has to choose which one of three cars to buy. The DS/AHP decision tree of the example is presented in Figure 1. The three alternatives (A, B, and C) are evaluated based on four criteria: price, fuel consumption, comfort, and style. For each criterion, the alternatives are grouped based on their comparable with respect to the criteria in question. For example, with respect to price, the alternatives A and B are roughly equally good (approximately the same price), but they are cheaper than alternative C. The decision alternatives for price include the set \{A, B\}, and \Theta representing all alternatives (frame of discernment) instead of \{A, B\} and C, because in DS/AHP all pairwise comparisons are made against the \Theta-set. The idea is that instead of comparing alternatives between each other, the DM has to identify favourable alternatives - with respect to certain criteria - from the frame of discernment. In AHP, a 9-point scale (1-9) for the pairwise comparisons is used. In this example a 6-point scale (1-6) is used instead. Table 1 presents the used scale values and their linguistic meanings. Notice that numerical rating 1 is not present, because the DM only needs to distinguish sets of alternatives that are preferred over the frame of discernment.

<table>
<thead>
<tr>
<th>Focus</th>
<th>Best Car</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criteria</td>
<td>Price</td>
</tr>
<tr>
<td>Decision Alternatives</td>
<td>{A, B}</td>
</tr>
</tbody>
</table>

Figure 1: DS/AHP decision tree for the example of buying a car (Beynon et al., 2000).

After the decision tree is set up, the weights of criteria have to be defined. The weights for the criteria are obtained using the pairwise comparison method.
Table 1: Favourability scale.

<table>
<thead>
<tr>
<th>Opinion</th>
<th>Numerical rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely favourable</td>
<td>6</td>
</tr>
<tr>
<td>Strongly to extremely</td>
<td>5</td>
</tr>
<tr>
<td>Strongly favourable</td>
<td>4</td>
</tr>
<tr>
<td>Moderately to strongly</td>
<td>3</td>
</tr>
<tr>
<td>Moderately favourable</td>
<td>2</td>
</tr>
</tbody>
</table>

as in AHP (Beynon et al., 2000). The weights for this example are: price: 0.3982, fuel: 0.0851, comfort: 0.2159 and style: 0.2988. The DM also has to make pairwise comparisons between groups of decision alternatives and the frame of discernment, from which a knowledge matrix is formed for each criterion. The knowledge matrix for comfort is presented in Table 2. After the comparisons are made, the knowledge matrices are multiplied by the weights for criteria. After this, priority values are obtained for groups of alternatives and the frame of discernment using the eigenvector method. The priority values are presented in Table 3.

Table 2: Knowledge matrix for comfort.

<table>
<thead>
<tr>
<th>Comfort</th>
<th>A</th>
<th>B, C</th>
<th>A, B, C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>B, C</td>
<td>0</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>A, B, C</td>
<td>1/4</td>
<td>1/6</td>
<td>1</td>
</tr>
</tbody>
</table>

After the priority values have been obtained, they are combined using Dempster’s rule of combination. After combining the four sources of evidence - fuel, comfort, price and style - the bpa \( m_{car} \) is obtained as presented in Table 4. Applying \( m_{car} \) belief and plausibility values for subsets of \( \Theta \) can be calculated using (10), and the DM makes a decision based on these values. DS/AHP also measures uncertainty of the results expressed as \( m_{car}(\Theta) \), which is 5.95%. For brevity, we will not present the final results here. The interested reader should refer to (Beynon et al., 2000).

5 DS/AHP in SMAA

We will next present a novel method for using DS/AHP to collect and aggregate preference information from multiple DMs. In this method, we will use DS/AHP to obtain preference information and aggregate it to form weight intervals. The DS/AHP is applied in the method, but instead of performing pairwise comparisons between alternatives, the pairwise comparisons are performed between criteria. Another change is that the pairwise comparison matrices will not be weighted.
Table 3: Priority values for groups of alternatives and the frame of discernment ($\Theta$).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Alternative</th>
<th>Priority</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>A, B</td>
<td>0.7050</td>
<td>0.2950</td>
</tr>
<tr>
<td></td>
<td>$\Theta$</td>
<td>0.2034</td>
<td></td>
</tr>
<tr>
<td>Fuel</td>
<td>B</td>
<td>0.2034</td>
<td>0.7966</td>
</tr>
<tr>
<td></td>
<td>$\Theta$</td>
<td>0.7966</td>
<td></td>
</tr>
<tr>
<td>Comfort</td>
<td>A</td>
<td>0.2466</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B, C</td>
<td>0.3259</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Theta$</td>
<td>0.4275</td>
<td></td>
</tr>
<tr>
<td>Style</td>
<td>A</td>
<td>0.3608</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.1891</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Theta$</td>
<td>0.4501</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: The bpa $m_{car}$ after combining all evidences.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>A, B</th>
<th>B, C</th>
<th>$\Theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{car}$</td>
<td>0.4665</td>
<td>0.2290</td>
<td>0.0482</td>
<td>0.1423</td>
<td>0.0545</td>
<td>0.0595</td>
</tr>
</tbody>
</table>

by constants but instead combined directly. The combination is executed using the Yager’s rule of combination, because it does not distribute the uncertainty evenly between criteria, but adds it to the $\Theta$-set instead. After the combination, the intervals for weights are obtained as the interval $[\text{Bel},\text{Pls}]$ for the sums of all sets of criteria.

The reason for choosing the Yager’s rule is that SMAA is designed for decision problems with inaccurate or obsolete preference information, and it performs the analysis based on all possible preferences. Because of this, it is preferable to use rather too large intervals for the weights than too small. The Dempster’s rule removes the conflicts by distributing the weight of conflict, while the Yager’s rule adds it to the uncertainty in the combined bpa.

The following small example illustrates the use of DS/AHP for collecting and aggregating preference information in SMAA. The decision making problem consists of 3 cars from which a group of DMs have to choose one. The cars are evaluated based on 3 criteria: style, price and fuel consumption. The normalized criteria measurements for the alternatives are presented in Table 5. The criteria are Gaussian distributed with standard deviation 10% of the mean value. Based on the criteria measurements, choosing the car depends completely on the preferences of the DMs; if price is preferred over the other criteria, then alternative B should be chosen, etc. Without any preference information an informed decision can not be
made.

Table 5: Criteria measurements for alternative cars.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Style</th>
<th>Price</th>
<th>Fuel</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$2.0 \pm 0.2$</td>
<td>$1.0 \pm 0.1$</td>
<td>$1.0 \pm 0.1$</td>
</tr>
<tr>
<td>B</td>
<td>$1.0 \pm 0.1$</td>
<td>$2.0 \pm 0.2$</td>
<td>$1.0 \pm 0.1$</td>
</tr>
<tr>
<td>C</td>
<td>$1.0 \pm 0.1$</td>
<td>$1.0 \pm 0.1$</td>
<td>$2.0 \pm 0.2$</td>
</tr>
</tbody>
</table>

There are three DMs present, who first recognize the sets of criteria that rise over the frame of discernment. After the recognition, the DMs perform pairwise comparisons of the distinguished sets against the frame of discernment. The results of these are presented in Tables 6, 7 and 8. These knowledge matrices show that DM 1 thinks that style is the most important criterion, but gives some weight to price and fuel consumption as well. DM 2 does not think that other criteria but fuel consumption are important, but neither thinks that the goodness of a car can be measured based on that alone. DM 3 thinks that style and price are both quite important criteria.

Table 6: Knowledge matrix for DM 1.

<table>
<thead>
<tr>
<th>DM 1</th>
<th>Style</th>
<th>Price, Fuel</th>
<th>Θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Style</td>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Price, Fuel</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Θ</td>
<td>1/2</td>
<td>1/6</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 7: Knowledge matrix for DM 2.

<table>
<thead>
<tr>
<th>DM 2</th>
<th>Fuel</th>
<th>Θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Θ</td>
<td>1/5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 8: Knowledge matrix for DM 3.

<table>
<thead>
<tr>
<th>DM 3</th>
<th>Style, Price</th>
<th>Θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Style, Price</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Θ</td>
<td>1/4</td>
<td>1</td>
</tr>
</tbody>
</table>

After this we compute the eigenvectors as in AHP (Saaty, 1980), and obtain
the following bpa’s:

\[ m_1(Style) = 0.63733, \ m_1(\{Price, Fuel\}) = 0.21244, \ m_1(\Theta) = 0.15022, \]
\[ m_2(Fuel) = 0.83333, \ m_2(\Theta) = 0.16667, \]
\[ m_3(\{Style, Price\}) = 0.8, \ m_3(\Theta) = 0.2. \]

Combining these using the Yager’s rule of combination (12) results in the following bpa:

\[ m(Style) = 0.106222, \ m(Price) = 0.028326, \ m(Fuel) = 0.060444, \]
\[ m(\{Style, Price\}) = 0.444919, \ m(\{Style, Fuel\}) = 0, \ m(\{Price, Fuel\}) = 0.007081, \]
\[ m(\Theta) = 0.353007. \]

From this bpa we can calculate the belief and plausibility values using (13) and (14). Calculated belief and plausibility values are presented in Table 9. Next we

<table>
<thead>
<tr>
<th>X</th>
<th>Bel(X)</th>
<th>Pls(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{Style}</td>
<td>0.11</td>
<td>0.90</td>
</tr>
<tr>
<td>{Price}</td>
<td>0.03</td>
<td>0.83</td>
</tr>
<tr>
<td>{Fuel}</td>
<td>0.06</td>
<td>0.42</td>
</tr>
<tr>
<td>{Style, Price}</td>
<td>0.58</td>
<td>0.94</td>
</tr>
<tr>
<td>{Style, Fuel}</td>
<td>0.17</td>
<td>0.97</td>
</tr>
<tr>
<td>{Price, Fuel}</td>
<td>0.10</td>
<td>0.89</td>
</tr>
<tr>
<td>{Style, Price, Fuel} (\Theta)</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

define constraints for sets of weights using the obtained belief and plausibility:

\[ 0.11 \leq w_{Style} \leq 0.90, \]
\[ 0.03 \leq w_{Price} \leq 0.83, \]
\[ 0.06 \leq w_{Fuel} \leq 0.42, \]
\[ 0.58 \leq w_{Style} + w_{Price} \leq 0.94, \]
\[ 0.17 \leq w_{Style} + w_{Fuel} \leq 0.97, \]
\[ 0.10 \leq w_{Price} + w_{Fuel} \leq 0.89 \]

We executed SMAA analysis using criteria values presented in Table 5 and the preference information. SMAA runs were executed using 100000 Monte Carlo iterations. The rank acceptability indices are presented in Table 10. From the results the effect of preference information is evident. Alternative A, which has a larger value for the style criterion, has a large confidence factor and high value for the acceptability index (\(b_1\)). As the preferences of the DMs are already present in the model, A should be chosen as the car to buy. In contrast, alternative C, which has largest value for fuel criterion, obtains a relatively small confidence factor and large acceptability for the worst rank (\(b_3\)), so it should not be bought. The combined preferences thus emphasize style more than price, and price more than fuel.
Table 10: Confidence factors and rank acceptability indices of the alternatives.

<table>
<thead>
<tr>
<th>Alt</th>
<th>$p^c$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>93.60</td>
<td>50.76</td>
<td>28.60</td>
<td>20.64</td>
</tr>
<tr>
<td>B</td>
<td>87.87</td>
<td>38.81</td>
<td>30.39</td>
<td>30.80</td>
</tr>
<tr>
<td>C</td>
<td>35.84</td>
<td>10.44</td>
<td>41.01</td>
<td>48.56</td>
</tr>
</tbody>
</table>

Table 11: Central weight vectors of the alternatives.

<table>
<thead>
<tr>
<th>Alt</th>
<th>Style</th>
<th>Price</th>
<th>Fuel</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>56.88</td>
<td>22.00</td>
<td>21.11</td>
</tr>
<tr>
<td>B</td>
<td>26.77</td>
<td>52.74</td>
<td>20.49</td>
</tr>
<tr>
<td>C</td>
<td>34.96</td>
<td>30.83</td>
<td>34.21</td>
</tr>
</tbody>
</table>

6 Conclusions

SMAA methods have a history of successful real-life applications. The method of aggregating the preferences of multiple DMs may have a large impact on the results. In this paper we have presented a novel method to handle preference information from multiple DMs. The method is based on collecting the DMs’ preferences using the DS/AHP method, aggregating them using the Yager’s rule of combination and representing them as interval constraints for weights in SMAA. We demonstrated the method using a small car-selection problem.

We are currently applying the method in a real-life problem. Future work should address the theoretical properties of the suggested method, as well as tests with larger problems.

References


